An absolute calibration method for displacement sensors

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Abstract

In the present study, a new in situ absolute calibration method for a displacement sensor is proposed, and a calibration system is developed. This new method is capable of determining not only the linearity error but also the mean sensitivity (inclination of linear calibration line) on the base of the wavelength. The new measurement system consists of a compact laser interferometer and a previously developed in situ calibration system. The laser interferometer is used only to determine the necessary displacement shift quantity with an integer multiple of half wavelengths of the laser light source. Using this known displacement shift quantity, the mean sensitivity and the linearity error of a displacement sensor can be determined absolutely. The accuracy of the proposed method depends only on the stabilities of the calibrated displacement sensor itself and the wavelength of the laser light source. A capacitance-type non-contact displacement sensor was calibrated successfully to the limit of stability of this sensor, which is determined from the signal-to-noise ratio.

Keywords: Metrology; Absolute calibration; Displacement sensor; Sensitivity; Non-linearity

1. Introduction

Displacement sensors have long been used in ultra-precision measurements of geometrical quantities such as positioning, vibration and profile measurement of shafts and surfaces. Using special detection techniques, displacement sensors can achieve extremely high resolution [1]. However, the accuracy of displacement sensors is affected by inherent systematic errors, such as the linearity error (deviation from a linear calibration line) of the sensor output. In particular, the linearity error is much larger than the resolution level. For example, although some displacement sensors have a nanometer resolution with a maximum measurement range of 100 μm, the assured accuracy, due chiefly to the linearity error, is only 0.2% of the measurement range [1]. In addition, since the characteristics of the linearity error are not stationary, the displacement sensors used in ultra-precision measurements require frequent calibration in order to adjust for the environment in which they are used (in situ condition). However, several problems are associated with such calibration. Such problems include the difficulty involved in obtaining a correct reference that has better than sub-nanometer resolution and how to perform the frequent calibration that is required for the displacement sensors in the measurement system. In other words, these displacement sensors require in situ calibration.

The present authors have previously proposed an in situ self-calibration method of the linearity error.
for several geometrical sensors, such as a displacement sensor [2] and an angle sensor [3]. This method has also been applied to calibrate a scanning white light interferometer microscope [4]. The proposed method is based on the acquisition of the derivation function of the calibration curve, which is only slightly affected by the accuracy of the input quantity. This principle is elucidated analytically. Computer simulations are used to confirm that the linearity error, several percent of the measurement range, can be correctly calibrated using the proposed method. The accuracy of the proposed method depends only on the stability and resolution of the calibrated sensor. However, the proposed method is based on the premise that the mean sensitivity of the sensor be given beforehand; therefore, only the linearity error, the deviation from a linear calibration line, can be determined.

In the present study, a new in situ absolute calibration method for a displacement sensor is proposed, and a calibration system is developed. This new method can be used to determine not only the linearity error but also the mean sensitivity (the inclination of the linear calibration line) on the base of the wavelength. The new measurement system combines a compact laser interferometer and the previously developed in situ calibration system [2]. The laser interferometer is used only to determine the necessary displacement shift quantity with an integer multiple of half wavelengths of the laser light source. Using this known displacement shift quantity, the mean sensitivity and the linearity error of a displacement sensor can be determined absolutely.

In the present report, the principle of the proposed in situ absolute calibration for a displacement sensor and its calibration system are introduced. In addition, the feasibility of the proposed method is demonstrated experimentally. A capacitance-type non-contact displacement sensor is calibrated successfully to the limit of stability of this displacement sensor which is determined from the signal-to-noise ratio.

2. Absolute calibration principle

In general, calibration can be divided into two sections: mean sensitivity calibration and linearity error calibration [see Fig. 1(a)]. In the first section, the mean sensitivity, which is related to the inclination of the linear calibration line of a sensor, is determined. In addition, the linearity error, which is related to the deviation from the linear calibration line, is determined.

Fig. 1. Principle of absolute calibration: (a) output curve; (b) inverse function of output curve.
2.1. Principle

The principle of absolute calibration is based on the same concept employed in the in situ calibration of a displacement sensor [2], in which, rather than performing the converging calculation, the inverse function is calculated. The function \( f(x) \) shown in Fig. 1(a) is the output curve of a displacement sensor that can generally be expressed as the sum of a straight line having a slope of \( S_0 \) and the deviation function \( g(x) \) from the straight line. The function \( f(x) \) is expressed as follows:

\[
z = f(x) = S_0 \times x + g(x)
\]

Here, \( S_0 \) and \( g(x) \) represent the mean sensitivity and the linearity error, respectively. Let the output of the displacement sensor at sampling position \( x_i \) be \( m_i \), and that at position \( x_i + \Delta x \) (\( \Delta x \) is the displacement shift quantity) be \( m_{i+} \). If the inverse function of \( f(x) \) shown in Fig. 1(b) is given by:

\[
f^{-1}(z_i) = z_i / S_0 - h(z_i) \quad (i = 1, 2, \ldots, n)
\]

where \( n \) is the sampling number and \( h(z) \) is the inverse function of \( g(x) \). The derivative of \( h(z) \) at the discrete \( i \)th sampling point can be expressed by

\[
h'(z_i) = 1 / S_0 - \Delta x / \Delta m_i \quad (\Delta m_i = m_{i+} - m_i)
\]

Here, the necessary displacement shift quantity \( \Delta x \) can be determined using an integer multiple of half wavelengths of the laser light source. \( S_0 \) can be calculated by

\[
S_0 = (\sum \Delta m_i / \Delta x) / n
\]

and \( h(z) \) can be obtained by integrating \( h'(z_i) \).

2.2. Simulation results

Fig. 2 shows the simulation results. The measurement range of the calibrated displacement sensor is 100 \( \mu \)m. The displacement shift quantity \( \Delta x \) is 4 \( \mu \)m. The nominal value of mean sensitivity is 0.2 V \( \mu \)m\(^{-1}\) and its linearity error is a sinusoidal curve of zero-line symmetry with an amplitude of 200 nm. The accidental error having a deviation of \( \sigma = 2 \) nm is included in the sampling of the sinusoidal curve. The nominal sampling intervals are 0.5 \( \mu \)m, and 200 sampling data are obtained within the measurement range of the displacement sensor. In addition, a random value having a maximum value of \( \pm 0.2 \) \( \mu \)m is added to the scanning intervals in order to simulate the deflection of sampling intervals in actual application of this calibration method.

The residual error, which is the difference between the given linearity error and the obtained linearity error, is also shown in Fig. 2. The maximum residual error is approximately 4 nm, which is twice the amplitude of the noise. This result is similar to that obtained by the converging calculation method [2].

3. Calibration system

Fig. 3(a) shows the experimental apparatus for calibrating the output curve of a displacement sensor. The system shown in Fig. 3(a) consists of a calibrated displacement sensor, a compact laser interferometer, which is used only to provide the necessary displacement shift quantity with an integer multiple of half wavelengths of the laser light source, and a Z-axis stage, which is used to scan the measurement range of the displacement sensor. The displacement sensor is fixed to a column, and directed toward the center of the worktable in order to fit the light axis of the laser interferometer. The displacement sensor detects the displacement of the worktable, which is moved by a tube PZT. The laser
interferometer is placed on the Z-axis stage, which is driven by a stepping motor.

Fig. 3(b) shows another experimental apparatus for calibrating the output curve of a displacement sensor. In this case, an X-axis stage and an inclined flat surface for scanning the measurement range of the displacement sensor are used instead of the Z-axis stage. The inclined flat surface which is made from a gauge block is fixed to the worktable center of the laser interferometer, and the laser interferometer is placed on the X-axis stage, which is driven by a stepping motor. The inclined flat surface can produce a continuous displacement change in the Z-direction over the calibration range when the X-axis stage is driven along the X-direction.

3.1. Laser interferometer

Fig. 4 shows the construction and optics system of the developed laser interferometer. The incident light from a laser light source system (LDS) is split into two beams by a cube beam splitter (BS) inside the tube PZT (Φ 15.5 mm × 13.5 mm). One beam is reflected by a moving mirror that is fixed on the bottom of the worktable, and the other beam is reflected by a fixed mirror. The two beams are then reflected back to the BS, producing an interference fringe, which is received by a 2-dividing photodiode after passing through a bi-concave lens. When the worktable is moved by the tube PZT, the fringe moves according to the light distance change between the two reflected beams. The outputs at two points of phase difference π of the fringe are detected by the 2-dividing photodiode.

If x1 and x2 are the outputs of the 2-dividing photodiode, and Y is the output of the laser interferometer, then Y is defined by the following equation:

\[
Y = \frac{x_1 - x_2}{x_1 + x_2}
\]  

The differential detection method can be used to effectively eliminate intensity variations in the laser light source of the laser interferometer. Furthermore, because the optical path difference between two interference beams is as small as possible and the two interference beams inside the tube PZT will be influenced only slightly by the air turbulence, the stable output of interference fringes can be obtained. However, wavelength variations of the laser light source due to thermal drift cannot be eliminated by the differential detection method. Thus, in order to improve the stability of the wavelength, and thereby achieve higher accuracy, a thermal controller was added to the laser light source. The laser light source in Fig. 4(b) consists of a laser diode (10 mW, λ = 785 nm), a thermal sensor and a Peltier-element. The Peltier-element is used to regulate the detected temperature of the laser light source in order to suppress thermal drift in the laser light source.

The output of the laser interferometer shown in Fig. 4(c) is a sinusoidal voltage signal with good symmetry about the zero-line.
3.2. Signal processing system

Fig. 5 is a diagram of the signal processing system used to calibrate the displacement sensor. The output of the calibrated sensor is detected by a digital multimeter. The calibration data \( m_i \) of the output curve of the displacement sensor is sampled using the first sampling pulse signal from the laser interferometer after the Z-axis stage is moved by the stepping motor. The calibration data \( m_{i+1} \) corresponding to \( m_i \) is sampled continuously after the first sampling pulse signal. The sampling pulse signals are produced at zero-crossing points of a sinusoidal voltage signal from the laser interferometer while the tube PZT is moved. This sinusoidal voltage signal is produced by the outputs of the 2-dividing photodiode, which are converted by \( I-V \) converters and calculation circuits (addition, subtraction and division). The tube PZT is driven by a sawtooth voltage of 1-Hz frequency which is produced by a function generator after passing through an analog multiplexer and booster. The displacement quantity between neighboring pulses is \( \lambda/2 \) (\( \lambda \) is the wavelength of the laser light, and \( \lambda/2 \) is the period of the fringe). The necessary displacement shift quantity \( \Delta x \) for in situ absolute calibration is \( N \times \lambda/2 \) (\( N \) is an integer).

The proposed calibration system provides an extremely simple mechanical structure, and one important specific characteristic in this calibration system is that an extremely simple laser interferometer can be used because zero-crossing points of the interference fringes are necessary while the interpolation of the interference fringes is unnecessary. The laser interferometer in the calibration system is only
used to obtain the mean sensitivity with an integer multiple of wavelengths of the laser light source, so that the calibration accuracy of the system is not affected by the unknown non-linearity motion of the tube PZT. Another important specific characteristic is that the calibration accuracy of the system is not associated with the sampling points theoretically, that is, it is also not affected by the tilting of the Z-axis stage [see Fig. 3(a)] or the X-axis stage [see Fig. 3(b)] which are only used to produce a continuous displacement change in the Z-direction.

4. Experimental results

The calibrated sensor is a capacitance non-contact type displacement sensor having a ±50 μm measurement range. The nominal value of mean sensitivity of the displacement sensor as reported by the manufacturer (ADE3046-A01 Console, 2102/2001 Microsense) is 0.2 V μm⁻¹.

Fig. 6 shows the thermal drift of the displacement sensor. The stepping motor and tube PZT were switched on and maintained in a stationary state. The measurement period was 10 min. During this period, the variation in the environmental temperature of a small simple thermo-insulation box containing the displacement sensor and the calibration apparatus was less than 0.01°C. The high-frequency component of the thermal drift was approximately 3 nm min⁻¹, and the low-frequency component was approximately 2 nm min⁻¹. The main cause of the high-frequency drift is thought to be electrical noise. The main causes of the low-frequency drift are variation of the mechanical deformation of the structure of the calibration system and the creep of the tube PZT. However, the Δnᵢ term in Eqs. (3) and (4) is only slightly affected by low-frequency drift. Therefore, the resolution (3 nm) of the displacement sensor in the calibration environment was estimated from the thermal drift value.

Fig. 7 shows the two output curves of the calibrated displacement sensor obtained on different
days. The scanning intervals of the Z-axis stage are approximately 0.5 μm, and 193 calibration data were sampled within a 100 μm measurement range of the displacement sensor. This system requires approximately 5 min to perform one calibration. The necessary displacement shift quantity Δx for in situ absolute calibration is $10 \times \lambda/2$. The wavelength $\lambda$ of the laser light is 784.73 nm as measured using a HP70950 optical spectrum analyzer. In Fig. 7, the dotted line is the nominal value of the mean sensitivity of the calibrated displacement sensor, and the obtained mean sensitivity values of two repeated calibrations are approximately 1% less than the nominal value.

Fig. 8(a) shows the linearity error of the calibrated displacement sensor and the stability of the calibration results of the linearity error. The maximum linearity error was approximately 200 nm. The maximum deviation in the difference between the two results calibrated on different days was approximately ±10 nm. These results are worse than those obtained using the thermal drift test shown in Fig. 6. This may be due to instabilities in the calibration system.

Fig. 8(b) also shows the linearity errors of the calibrated displacement sensor obtained by the other experimental apparatus shown in Fig. 3(b) in order to investigate the reproducibility of calibration results. Repeatability between two separate calibrations is approximately ±10 nm. Averaging the results of each scanning method and comparing the two averaged values, the reproducibility error shown in Fig. 8(c) is approximately ±12 nm.

Fig. 9 shows the calibration results of another displacement sensor. This sensor is the capacitance non-contact type sensor having a 500 μm measurement range. The nominal value of mean sensitivity of the displacement sensor as reported by the manufacturer (ADE3820 Micro-sense) is 0.04 V μm$^{-1}$. The specimen is this sensor with a resolution of

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**Fig. 8.** Calibration results of the linearity error: (a) result for system 1; (b) result for system 2; (c) comparison of the two results.
approximately 10 nm and a stability of approximately 16 nm min\(^{-1}\). The scanning intervals of the Z-axis stage are approximately 4 \(\mu\)m, and 126 calibration data were sampled within a 500 \(\mu\)m measurement range of the displacement sensor. The necessary displacement shift quantity \(\Delta x\) for in situ absolute calibration is \(10 \times \lambda/2\). The maximum linearity error was approximately 600 nm. The maximum deviation between the two results calibrated on different days was approximately 50 nm. The obtained mean sensitivity is approximately 0.5\% less than the nominal value of the calibrated displacement sensor.

5. Error analysis of the calibration system

In the calibration system, the displacement error of the Z-direction will arise if the tube PZT does not move the worktable parallel to the light axis of the interferometer. The displacement error of the calibration system shown in Fig. 3(a) can be calculated as follows:

\[
\Delta Z = L_p \times (1 - \cos \alpha)
\]  

(6)

where \(L_p\) is the scanning distance of the tube PZT \((L_{pMAX} = 4 \, \mu\text{m})\), \(\alpha\) is the tilting angle of the tube PZT to the light axis of the interferometer, and \(\Delta Z\) is the displacement error of the Z-direction that arises due to the tilting angle \(\alpha\). Here this displacement error is inherent when the calibration is usually based on comparative measurement with an interferometer, and it is a second-order error, which is too small to be taken into account.

The displacement error of the calibration system shown in Fig. 3(b) can be calculated as follows:

\[
\Delta Z = L_x \times \tan \Delta \beta
\]  

(7)

where \(L_x\) is the distance \((L_{xMAX} = 10 \, \text{mm})\) between the axis of the tube PZT and the measuring axis of the calibrated displacement sensor when the X-axis stage is driven along the X-direction. \(\Delta \beta\) is the tilting angle variation of the worktable that arises due to the tilting angle variation of the tube PZT at the sampling positions. Fig. 10 shows the tilting angle variations (pitching and yawing) of the tube PZT over a scanning range of 4 \(\mu\)m as measured using a Nikon Dual-axis Photoelectric autocollimator. The measurement time was 45 s. The sampling period was 0.015 s, and 3000 measurement data were sampled. The tube PZT is driven by a triangular voltage of 1-Hz frequency which is produced by a function generator as in the actual calibration experiments. In Fig. 10, the maximum tilting angle variations of the pitching and yawing of the tube PZT are approximately 0.2 s (arc) over the whole scanning range.

\(\Delta Z\) in Eq. (7) is the first-order error, the larger the distance \(L_x\), the larger the displacement error that arises due to the tilting angle variations of the worktable. For example, when \(L_{xMAX} = 10 \, \text{mm} \) and \(\Delta \beta_{MAX} = 0.2 \, \text{s (arc)}\), \(\Delta Z\) is approximately 10 nm.

Fig. 11 shows photographs of the developed calibration system and the laser interferometer. The
Fig. 11. Photographs of the calibration system and laser interferometer: (a) calibration system; (b) laser interferometer.
size of the laser interferometer is 90 mm × 90 mm × 70 mm.

6. Conclusions

The results of the present research are summarized as follows:

1. A new in situ absolute calibration method is proposed for calibrating the output curve of a displacement sensor. The effectiveness of the method was confirmed experimentally.
2. The accuracy of the method depends only on the stabilities of the calibrated displacement sensor itself and the wavelength of the laser light source.
3. A capacitance-type non-contact displacement sensor was calibrated successfully, and the maximum repeatability error and reproducibility error of the calibration curve were approximately ±0.01% and ±0.012% of the measurement range of the displacement sensor, respectively.

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